Nonlinear and nonsmooth dynamics of discretely defined system of rigid bodies with unilateral contacts

(Nelinearna in nezvezna dinamika sistema diskretno definiranih togih teles z enostranskimi kontakti)

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October, 20^{th} 2005 - PhD Thesis Defence

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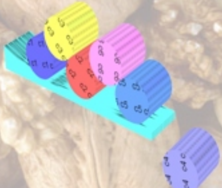
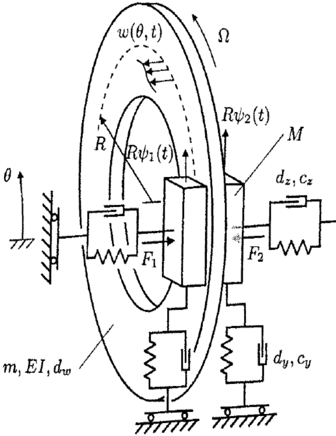
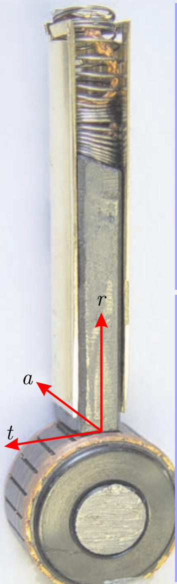
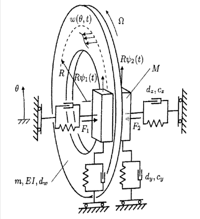



$w(\theta, t)$
Ω
$R\psi_2(t)$
M
R
$R\psi_1(t)$
θ
d_z, c_z
F_1
F_2
m, EI, d_w
d_y, c_y

Dynamics of systems with focus on contact surfaces

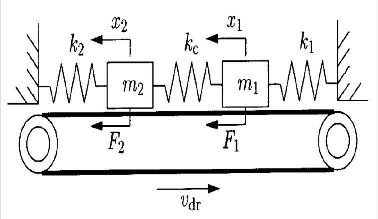


Heilig *et al.*



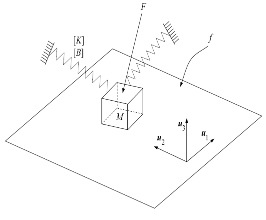
Nonlinear Dynamics (2003)

Vrande *et al.*



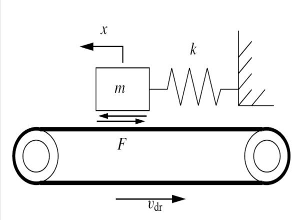
Nonlinear Dynamics (1999)

Cho and Barber



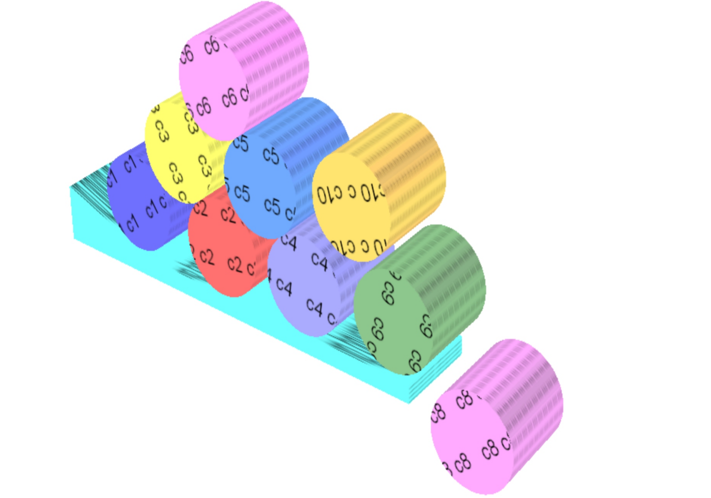
Proc. Royal Soc. Lond. A (1999)

Leine *et al.*

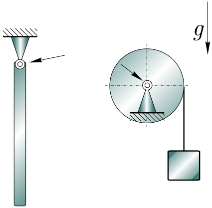


Nonlinear Dynamics (1998)

How?



Equations of motion for systems with bilateral contacts

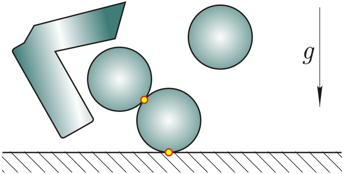


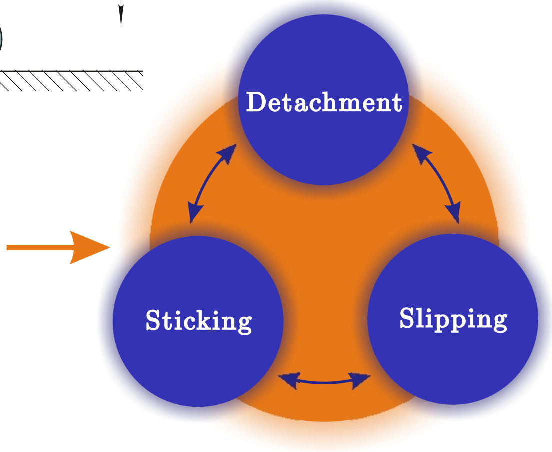
- N rigid bodies with f degrees of freedom:

$$\mathbf{M}(\mathbf{q}, t)\,\ddot{\mathbf{q}} - \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{0} \quad \in \mathbb{R}^f$$

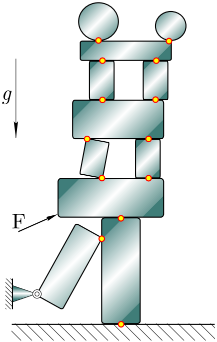
$\mathbf{M}$ - mass matrix,
$\mathbf{q}$ - vector of generalized coordinates,
$\mathbf{h}$ - vector of generalized active forces.

Unilateral contacts





Impact-free case



Number of possible solutions:

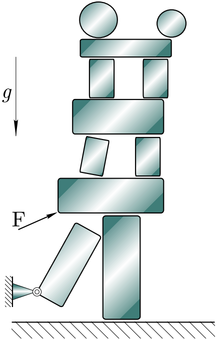
$$4^{n_{\mathrm{N}}} \approx 67\,\mathrm{mio.}$$

$$n_{\mathrm{N}} = 13$$

How to avoid time-consuming solution search and rebuilding of the set of generalized coordinates?

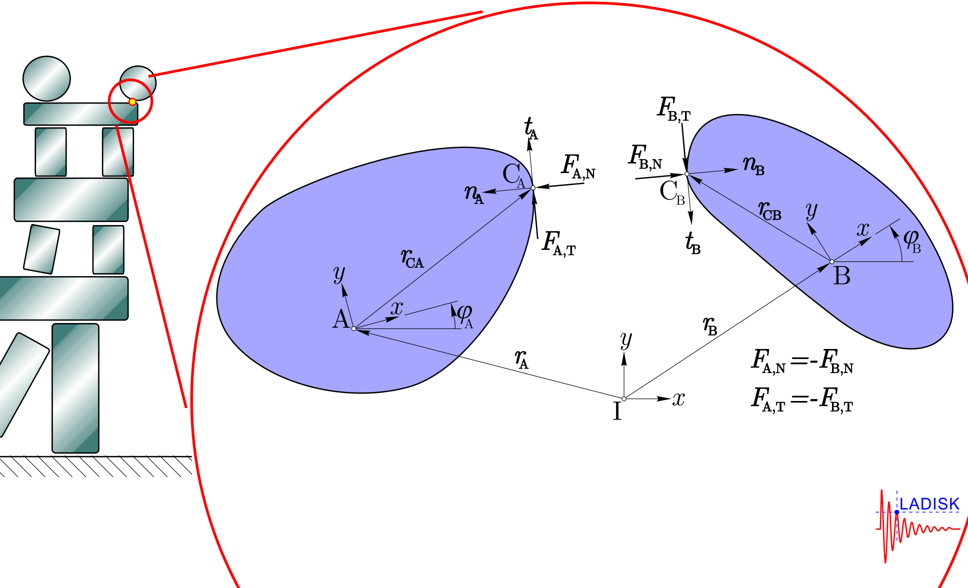
F. Pfeiffer and C. Glocker. *Multibody Dynamics with Unilateral Contacts.* Editors: A.H. Nayfeh and A.V. Holden, John Wiley & Sons, 1996.

Equations of motion of unilateral-contact free state



$$\mathbf{M}(\mathbf{q}, t)\,\ddot{\mathbf{q}} - \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, t) = \mathbf{0} \quad \in \mathbb{R}^f$$

Contact forces



Contact forces in generalized space

- Contact force on body A in generalized space (normal direction):

$$\mathbf{Q}^{\mathrm{c}}_{\mathrm{A,N}} = \left(\frac{\partial\, {}_{\mathrm{I}}\mathbf{r}_{\mathrm{C_A}}}{\partial\, \mathbf{q}}\right)^{\mathrm{T}} \mathbf{F}_{\mathrm{A,N}} = \mathbf{J}^{\mathrm{T}}_{\mathrm{C_A}} \cdot {}_{\mathrm{I}}\mathbf{n}_{\mathrm{A}} \cdot \lambda_{\mathrm{N}}.$$

- Sum of contact forces on body A and body B (normal direction):

$$\mathbf{Q}^{\mathrm{c}}_{\mathrm{N}} = \left(\mathbf{J}^{\mathrm{T}}_{\mathrm{C_A}}\, {}_{\mathrm{I}}\mathbf{n}_{\mathrm{A}} + \mathbf{J}^{\mathrm{T}}_{\mathrm{C_B}}\, {}_{\mathrm{I}}\mathbf{n}_{\mathrm{B}}\right) \lambda_{\mathrm{N}} = \mathbf{w}_{\mathrm{N}}\, \lambda_{\mathrm{N}},$$

- Equation of motion of a multibody system with n_{N} concurrent unilateral contacts:

$$\mathbf{M}\,\ddot{\mathbf{q}} - \mathbf{h} = \sum_{i \in I_{\mathrm{N}}} \mathbf{Q}^{\mathrm{C}}_{\mathbf{i}} \quad \Rightarrow \quad \mathbf{M}\,\ddot{\mathbf{q}} - \mathbf{h} - \sum_{i \in I_{\mathrm{N}}} (\mathbf{w}_{\mathrm{N}}\, \lambda_{\mathrm{N}} + \mathbf{w}_{\mathrm{T}}\, \lambda_{\mathrm{T}})_i = \mathbf{0} \quad \in \mathbb{R}^f.$$

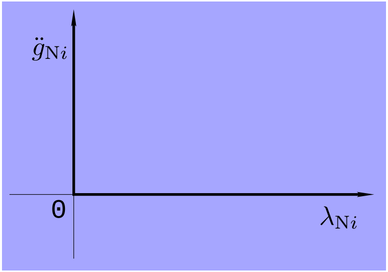
How do we find the missing values?

- Eq. of motion in matrix notation

$$\mathbf{M}\,\ddot{\mathbf{q}} - \mathbf{h} - \begin{pmatrix} \mathbf{W}_{\mathrm{N}} + \mathbf{W}_{\mathrm{G}}\,\overline{\overline{\mu}}_{\mathrm{G}} & \mathbf{W}_{\mathrm{H}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\lambda}_{\mathrm{N}} \\ \boldsymbol{\lambda}_{\mathrm{H}} \end{pmatrix} = \mathbf{0} \qquad \in \mathbb{R}^{f}.$$

($n_{\mathrm{N}} + n_{\mathrm{H}}$ unknown values.)

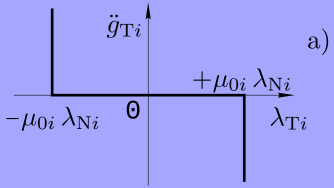
Complementarity in the normal direction



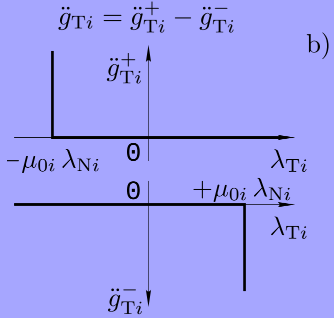
$$\ddot{g}_{\mathrm{N}i} \geq 0 \qquad \bigwedge \qquad \lambda_{\mathrm{N}i} \geq 0 \qquad\qquad \ddot{g}_{\mathrm{N}i}\, \lambda_{\mathrm{N}i} = 0$$

P. Lötstedt. *Mechanical systems of rigid bodies subject to unilateral constraints* SIAM J. Appl. Math., 1982.

Complementarity in the tangential direction



Physical



Mathematical

Stick/slip/detachment as a linear complementarity problem

$$\mathbf{y} = \mathbf{A}\,\mathbf{x} + \mathbf{b},$$

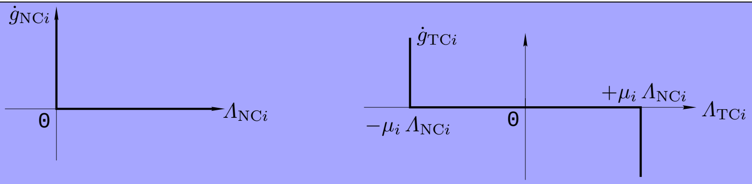
$$\mathbf{y} \geq 0, \qquad \mathbf{x} \geq 0, \qquad \mathbf{y}^{\mathrm{T}}\,\mathbf{x} = 0 \;(y_i\,x_i = 0).$$

$$\mathbf{x}, \mathbf{y} \in \mathbb{R}^{n_{\mathrm{N}}+4n_{\mathrm{H}}}.$$

$$\mathbf{y} \Leftrightarrow \ddot{\mathbf{g}}$$
$$\mathbf{x} \Leftrightarrow \boldsymbol{\lambda}$$

R. Cottle and G. Dantzig. *Complementary pivot theory of mathematical programming.* Linear Algebra and Appl., 1, 1968.

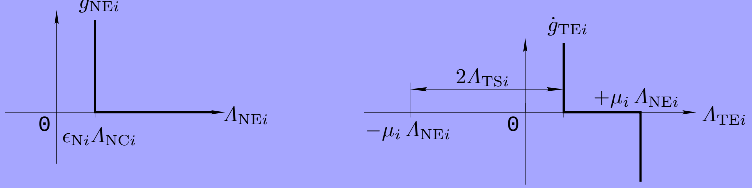
Impacts with friction



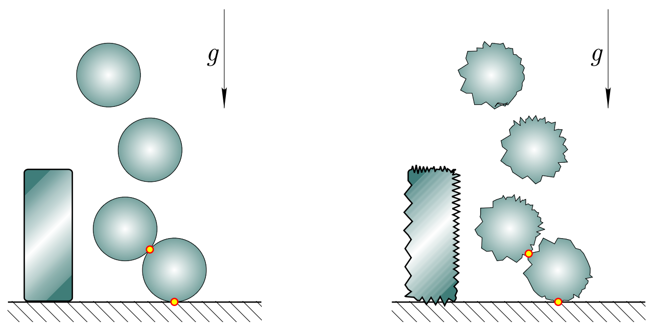
Compression phase

Poisson $\Downarrow$ law

Expansion phase

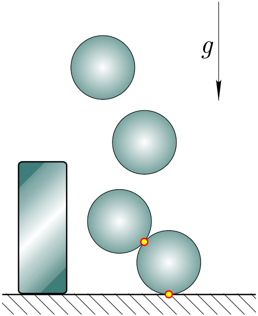


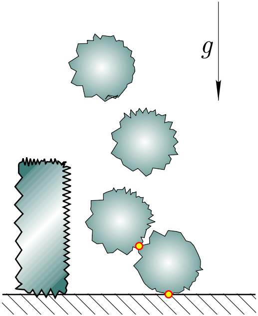
From analytically to discretely defined bodies



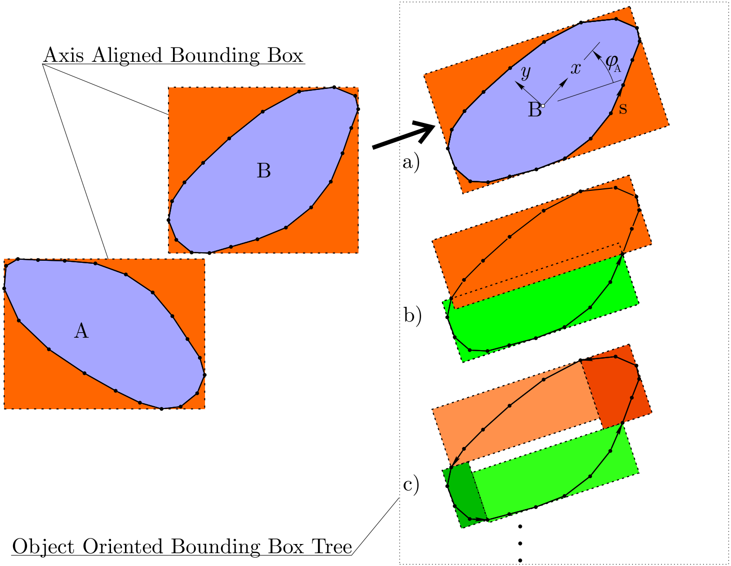
Extensions

- Collision detection.
- Automatic generation of kinematical properties of contact-points.
- Adaptable time-step with fixed-time-step output.
- Conformable contact dynamics (mechanical en. based).

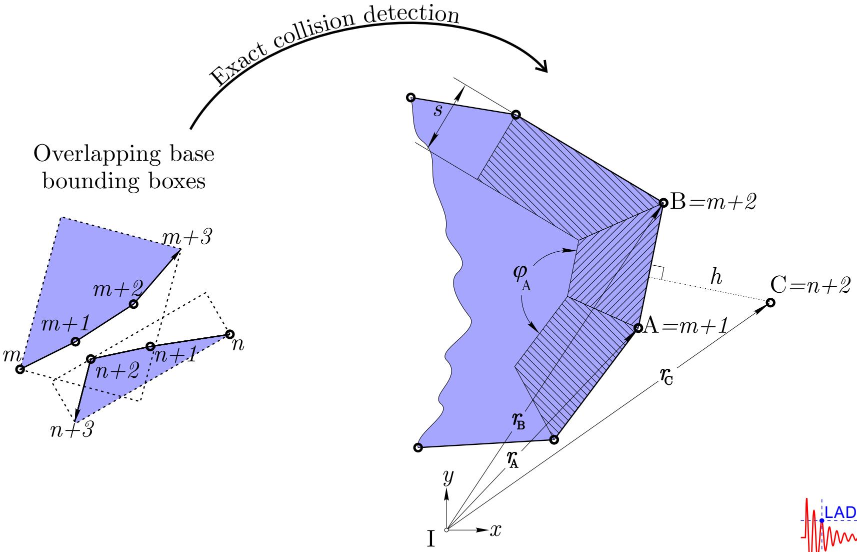




Collision detection

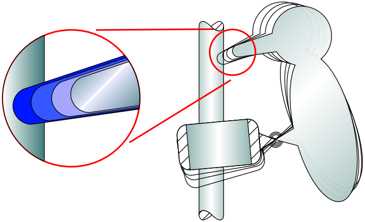


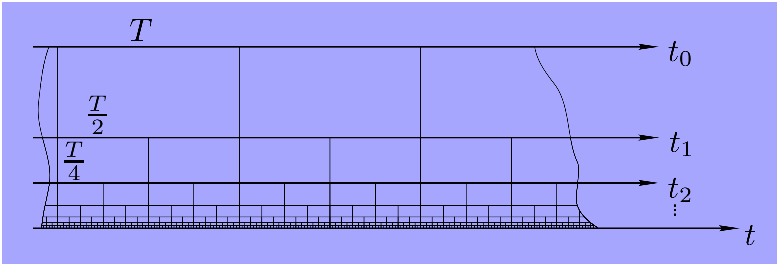
Exact collision detection



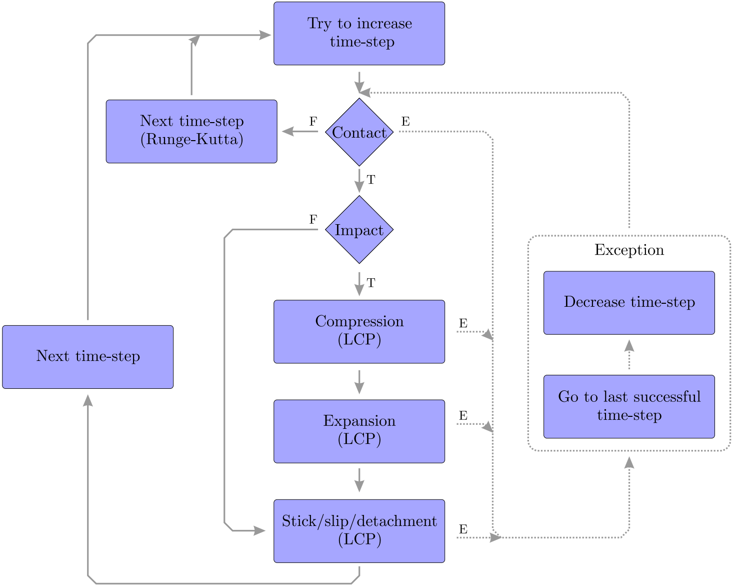
LADISK

Changing of the time-step

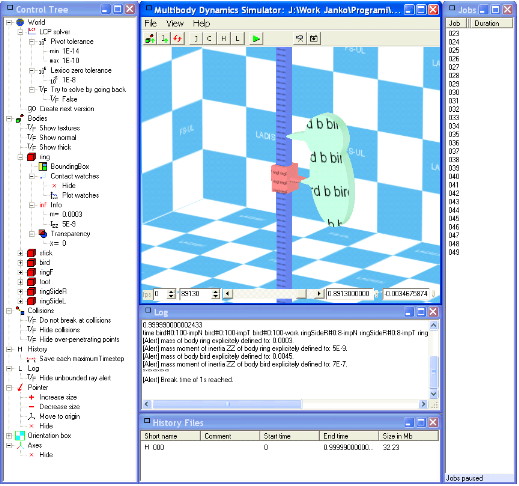




Flowchart


Try to increase time-step
Next time-step (Runge-Kutta)
F
Contact
E
T
F
Impact
T
Compression (LCP)
E
Expansion (LCP)
E
Stick/slip/detachment (LCP)
E
Next time-step
Exception
Decrease time-step
Go to last successful time-step

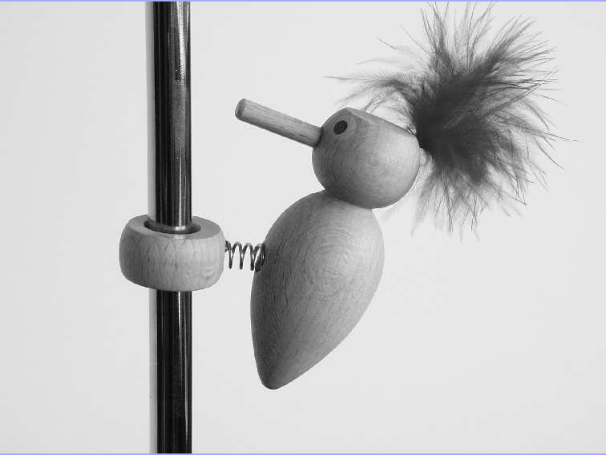
Implementation - Multibody Dynamics Simulator



- Packages for manipulating symbolic expressions written in *Mathematica*.
- *Object oriented multithreaded code* written in *Borland Delphi*.

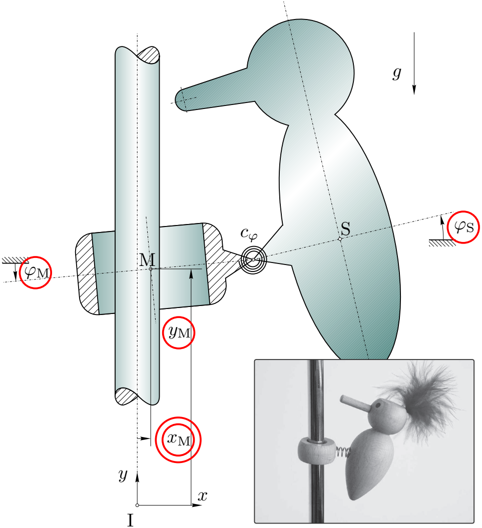
(Used open source resources: *GLScene*, *ParseExpr*.)

Multibody Dynamics
with Unilateral Contacts



F. Pfeiffer and C. Glocker, 1996

Mathematical model



$$\mathbf{q} = \begin{pmatrix} y_M \\ \varphi_M \\ \varphi_S \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} m_M + m_S & l_M\, m_S & l_G\, m_S \\ l_M\, m_S & J_M + l_M^2\, m_S & l_G\, l_M\, m_S \\ l_G\, m_S & l_G\, l_M\, m_S & J_S + l_G^2\, m_S \end{pmatrix}$$

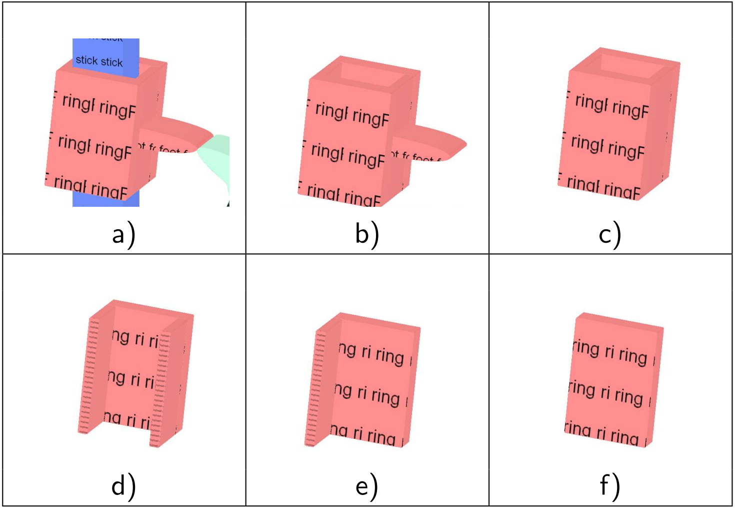
$$\mathbf{h} = \begin{pmatrix} -g\,(m_S + m_M) \\ -g\, l_M\, m_S + c_\varphi\, \varphi_S - c_\varphi\, \varphi_M \\ -g\, l_G\, m_S - c_\varphi\, \varphi_S + c_\varphi\, \varphi_M \end{pmatrix}$$

$$\bar{\mathbf{w}}_{N,1} = \begin{pmatrix} 0 \\ 0 \\ -h_S \end{pmatrix} \qquad \bar{\mathbf{w}}_{T,1} = \begin{pmatrix} 1 \\ l_M \\ l_G - l_S \end{pmatrix}$$

$$\bar{\mathbf{w}}_{N,2} = \begin{pmatrix} 0 \\ h_M \\ 0 \end{pmatrix} \qquad \bar{\mathbf{w}}_{T,2} = \begin{pmatrix} 1 \\ r_M \\ 0 \end{pmatrix}$$

$$\bar{\mathbf{w}}_{N,3} = \begin{pmatrix} 0 \\ -h_M \\ 0 \end{pmatrix} \qquad \bar{\mathbf{w}}_{T,3} = \begin{pmatrix} 1 \\ r_M \\ 0 \end{pmatrix}$$

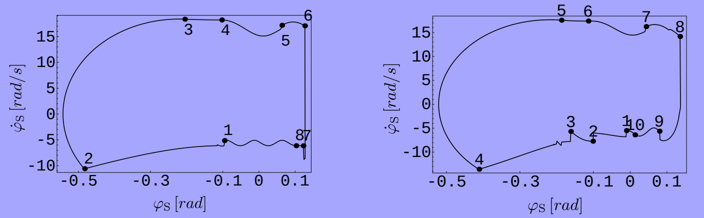
Virtual bodies



a) b) c)
d) e) f)

Simulation of 3 and 4 DOF models

Phase plots for φ_S



3DOF 4DOF

Published in: *Proc. IMechE - Part C: J. Mech. Eng. Science*

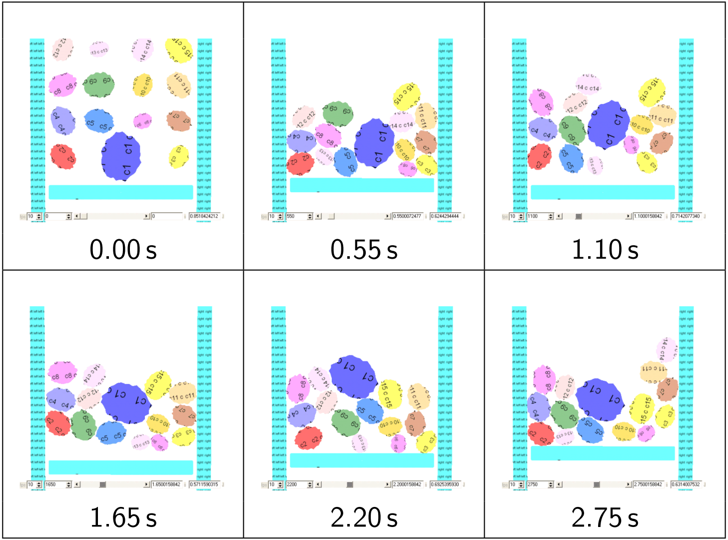
SLAVIČ, Janko, BOLTEŽAR, Miha. Nonlinearity and non-smoothness in multi body dynamics: application to woodpecker toy. *In Press*.

Granular materials:

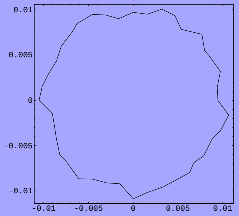
The brazil nut effect – in reverse

T. Shinbrot, *Nature* 2004

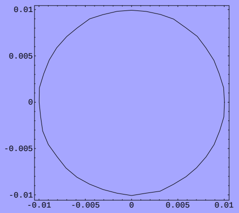
What is it?



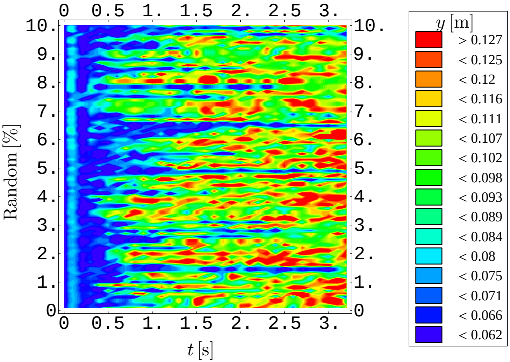
Rate of surface asperity influence



Random[8%]



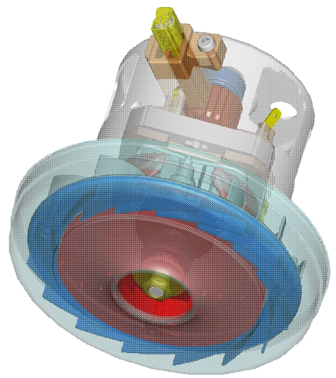
Random[0%]

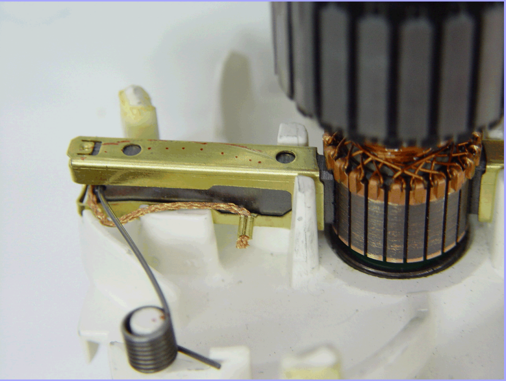


LADISK

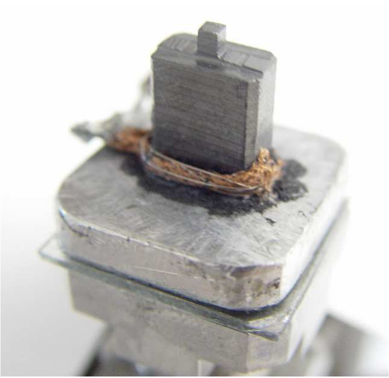
Sliding contact of a graphite pin

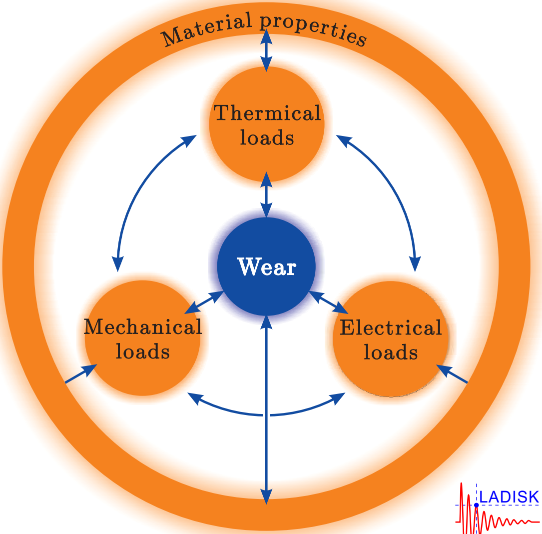
Domel d.d.





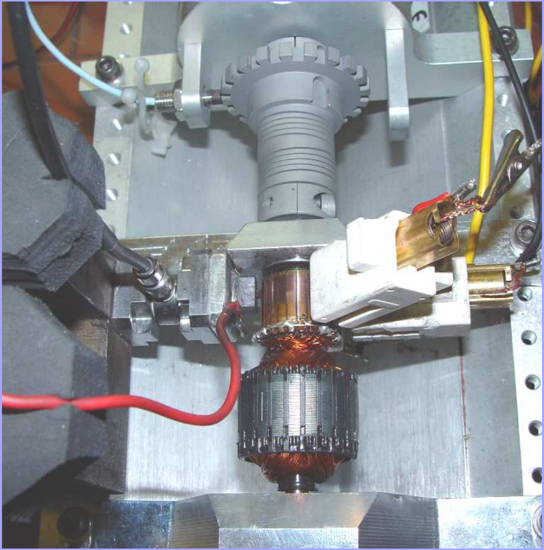
What defines the dynamics of the graphite pin?



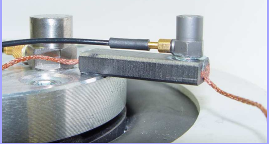


Experimental work

Coefficient of friction (T, I, s)

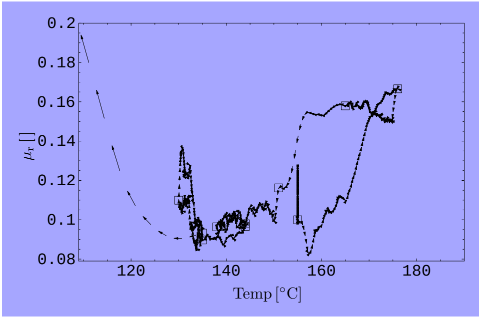


Stiffness (T, I), damp. (T)



Coeff. of restit. (T, I, h, v)

Instantaneous direct measurement of coefficient of friction



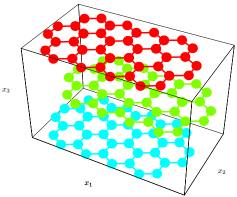
Key features of the *new method:*

- direct measurement
- dynamic forces
- temperature and current density influences
- possibility to check the prepositions:
  - COF independent of F_N,
  - axial direction negligible.

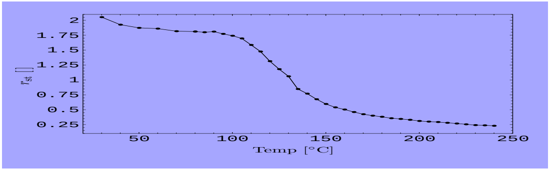
Published in: *Wear*

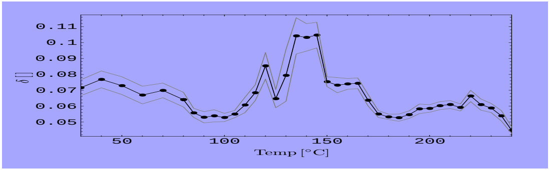
SLAVIČ, Janko, BOLTEŽAR, Miha. Measuring the dynamic forces to graphite-copper contact for variable temperature and current. *In Press.*

Stiffness of graphite/epoxy



$C_{11} = 17.7$ GPa, $C_{12} = 9.7$ GPa,
$C_{13} = 9.7$ GPa, $C_{33} = 17.7$ GPa,
$C_{44} = 4.0$ GPa.





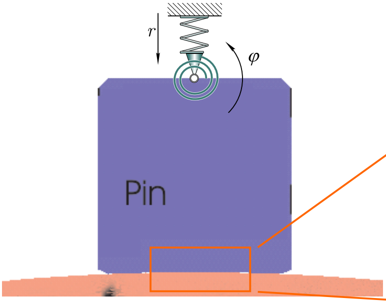
Published in: *Journal of Sound and Vibration*

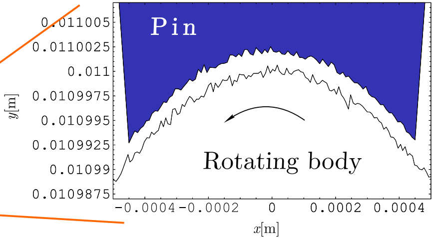
SLAVIČ, Janko, SIMONOVSKI, Igor, BOLTEŽAR, Miha. Damping identification transform: Application to real data. 2003.

Published in: *Mech. syst. signal process.*

BOLTEŽAR, Miha, SLAVIČ, Janko. Enhancements to the continuous wavelet transform for damping identifications on short signals. 2004.

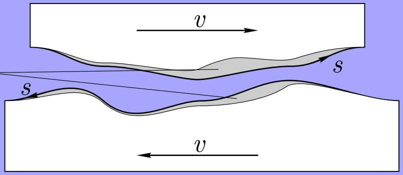
Conformable contact dynamics





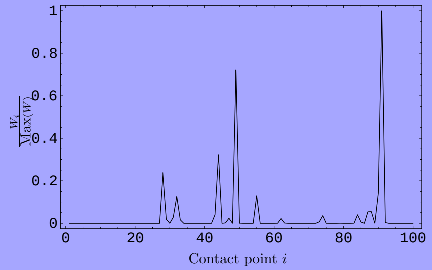
Wear ∝ W

Loss of mechanical energy at contacts

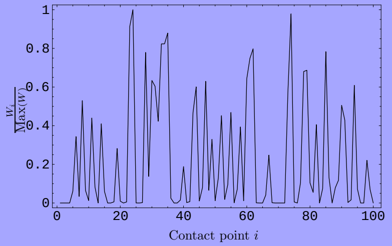


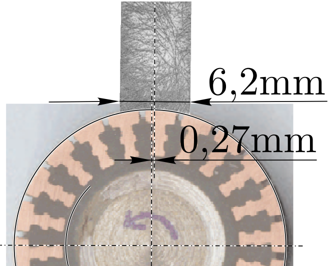
Re-shaping of the contact surface

Initial shape



After several re-shapings

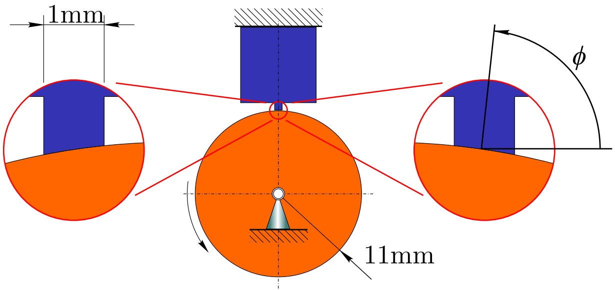


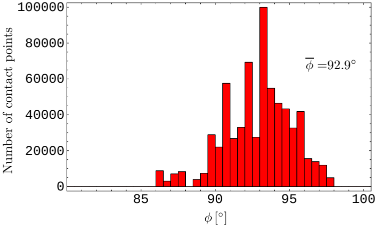


In agreement with experimental observations:

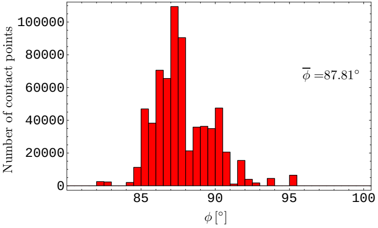
- increase of the radius of curvature,
- shift of the center of curvature.

Understanding the measured coefficient of friction





eccentricity $=-0.4\,\mathrm{mm}$



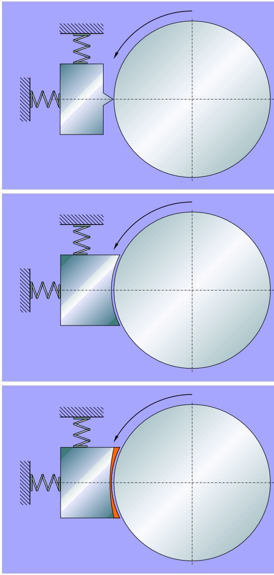
eccentricity $=+0.4\,\mathrm{mm}$

Contribution

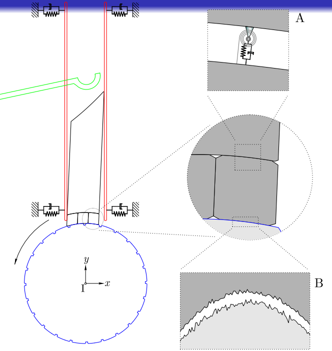
LADISK

Scientific contribution

- Extension of the Pfeiffer-Glocker formulation to discretely defined bodies.
- Introduction of virtual bodies.
- Formulation of loss of mechanical energy at contacts.
- Experimental method: measuring COF.
- Detailed analysis of friction of a sliding pin.
- Conformable contact dynamics.



Industrial application



Electric-motor-brush dynamics:

- 8 rigid bodies,
- 11 degrees of freedom,
- 46 parameters,
- *Monte Carlo:* 3400+ simulations.

Published in: *Journal of Mech. Eng. (Stroj. vestn.)*

SLAVIČ, Janko, Nastran, Miha, BOLTEŽAR, Miha. Modeling and analysis of the dynamics of an electric-motor brush. *In Press.*

Patent

New design for enhancing the quality of brush-commutator contact. *In preparation.*

Thank you for your attention

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of Ljubljana
Faculty
of Mechanical Engineering

Chair of Mechanics
Laboratory for Dynamics of Machines and Structures

LADISK